In a nutshell: Approximating solutions to higher-order initial value problems

Given an n^{th}-order initial-value problem (IVP)

$$\begin{aligned} y^{(n)}(t) &= f_1\left(t, y(t), y^{(1)}(t), \ldots, y^{(n-2)}(t), y^{(n-1)}(t)\right) \\ y(t_0) &= y_0 \\ y^{(1)}(t_0) &= y_0^{(1)} \\ &\vdots \\ y^{(n-2)}(t_0) &= y_0^{(n-2)} \\ y^{(n-1)}(t_0) &= y_0^{(n-1)} \end{aligned}$$

write this as

$$\mathbf{w}(t) = \begin{pmatrix} w_0(t) \\ w_1(t) \\ \vdots \\ w_{n-2}(t) \\ w_{n-1}(t) \end{pmatrix} = \begin{pmatrix} y(t) \\ y^{(1)}(t) \\ \vdots \\ y^{(n-n)}(t) \\ y^{(n-1)}(t) \end{pmatrix}, \ \mathbf{w}^{(1)}(t) = \mathbf{f}\left(t, \mathbf{w}(t)\right) = \begin{pmatrix} w_1(t) \\ w_2(t) \\ \vdots \\ w_{n-1}(t) \\ f\left(t, w_0(t), w_1(t), \ldots, w_{n-1}(t)\right) \end{pmatrix}, \text{ and } \mathbf{w}(t_0) = \begin{pmatrix} y_0 \\ y_0^{(1)} \\ \vdots \\ y_0^{(n-2)} \\ y_0^{(n-1)} \end{pmatrix} = \mathbf{w}_0 .$$

where we index $\mathbf{w}$ from 0 to $n - 1$ and not 1 to n so that the entry matches the derivative. This now defines a system of n 1st-order IVPs, for which we can use the previous techniques for approximation solutions to systems of 1st-order IVPs, the only difference is that we are now only interested in the first entry of the solution vector, for the first entry of $\mathbf{w}_k$ approximates $y(t_k)$.